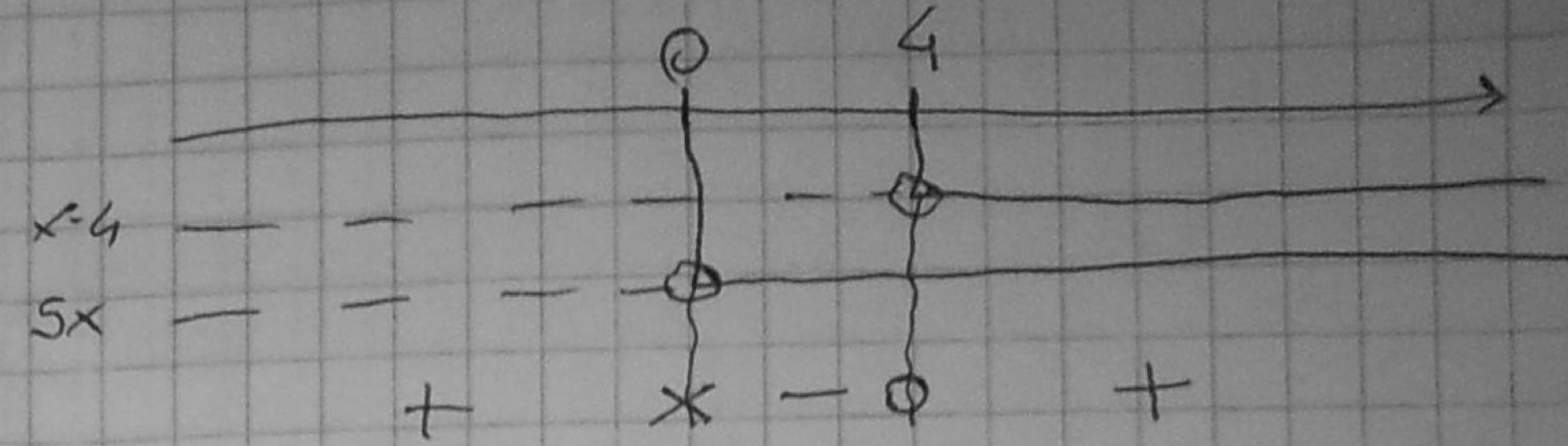
1) • $A = \left\{ x \in \mathbb{R} \mid \frac{x-4}{5x} \geq 0 \right\}$

Segno di $\frac{x-4}{5x}$ :




$A = (-\infty, 0) \cup [4, +\infty)$

• $B = \{ x \in \mathbb{R} \mid x-4 \neq 0 \} = (-\infty, 4) \cup (4, +\infty)$

• $C = (2,3)$

a) $A \nsubseteq B$, perchè $4 \in A$ ma $4 \notin B$

$C \subseteq B$, perchè $(2,3) \subseteq (-\infty, 4) \subset B$

b) $A \cup B = \mathbb{R}$, perchè $B = \mathbb{R} \setminus \{4\}$ e $4 \in A$.

$B \cup C = B$, perchè $C \subseteq B$

c) $A \cap B = A \setminus \{4\} = (-\infty, 0) \cup (4, +\infty)$, perchè intersecare con $\mathbb{R} \setminus \{4\}$ significa togliere il 4.

$A \cap C = C$, perchè $C \subseteq B$.

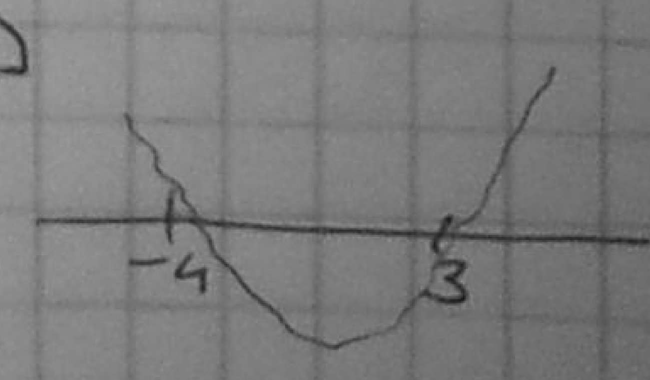
---

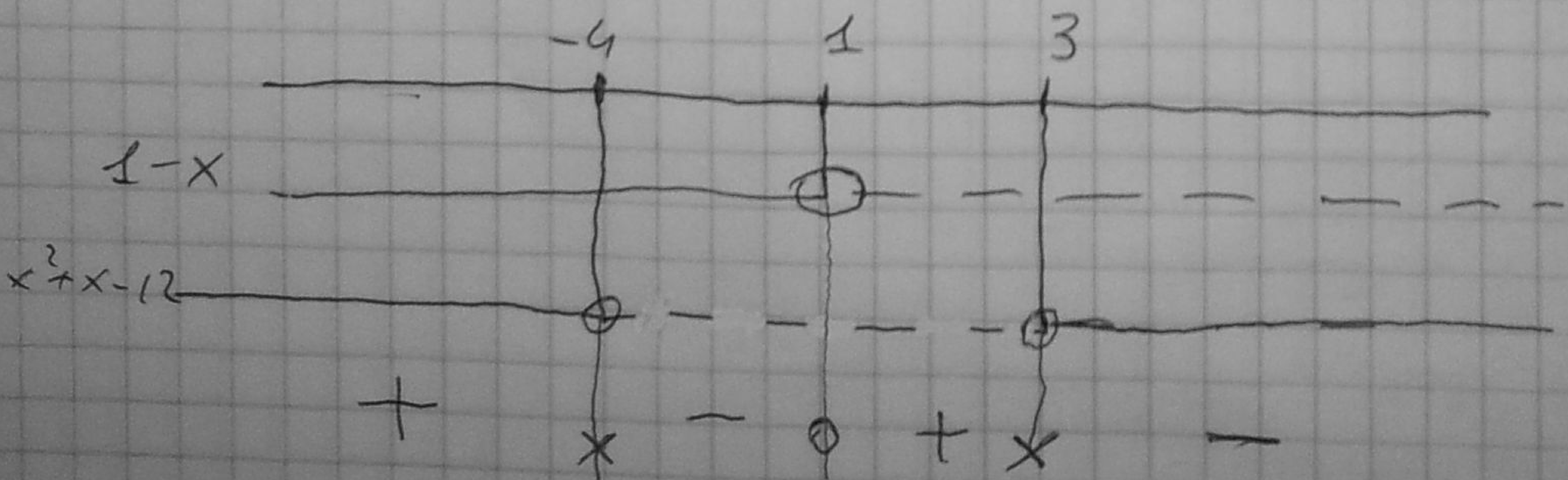
2) Segno di $\frac{1-x}{x^2+x-12}$ :

$x^2+x-12 = 0 \iff$

$x = \frac{-1 \pm \sqrt{49}}{2} = \begin{cases} 3 \\ -4 \end{cases}$

$\Rightarrow$




$\Rightarrow S = \left\{ \frac{1-x}{x^2+x-12} < 0 \right\} = (-4, 1) \cup (3, +\infty)$

3) $f(x) = \ln(x) - \ln(x-2)$

a) $D = \{x \in \mathbb{R} \mid x > 0\} \cap \{x \in \mathbb{R} \mid x > 2\} =$
$= (0, +\infty) \cap (2, +\infty) = (2, +\infty)$

b) •) $\ln(x) - \ln(x-2) > 0 \Rightarrow \ln\left(\frac{x}{x-2}\right) > 0$

$\Rightarrow \frac{x}{x-2} > 1 \Rightarrow \frac{x - x + 2}{x-2} > 0 \Rightarrow x > 2$

$\Rightarrow f(x) > 0$ sempre.

•) RISPOSTA ALTERNATIVA: $\ln(x)$ è una funzione CRESCENTE $\Rightarrow \ln(y) > \ln(x)$ se $y > x$

$\Rightarrow \ln(x) > \ln(x-2) \Rightarrow \ln(x) - \ln(x-2) > 0$

c) $\lim_{x \to 2^+} f(x) = \ln 2 - (-\infty) = +\infty \Rightarrow$ AS. VERT. dx $x = 2$

$\lim_{x \to +\infty} f(x) = +\infty - \infty = FI$

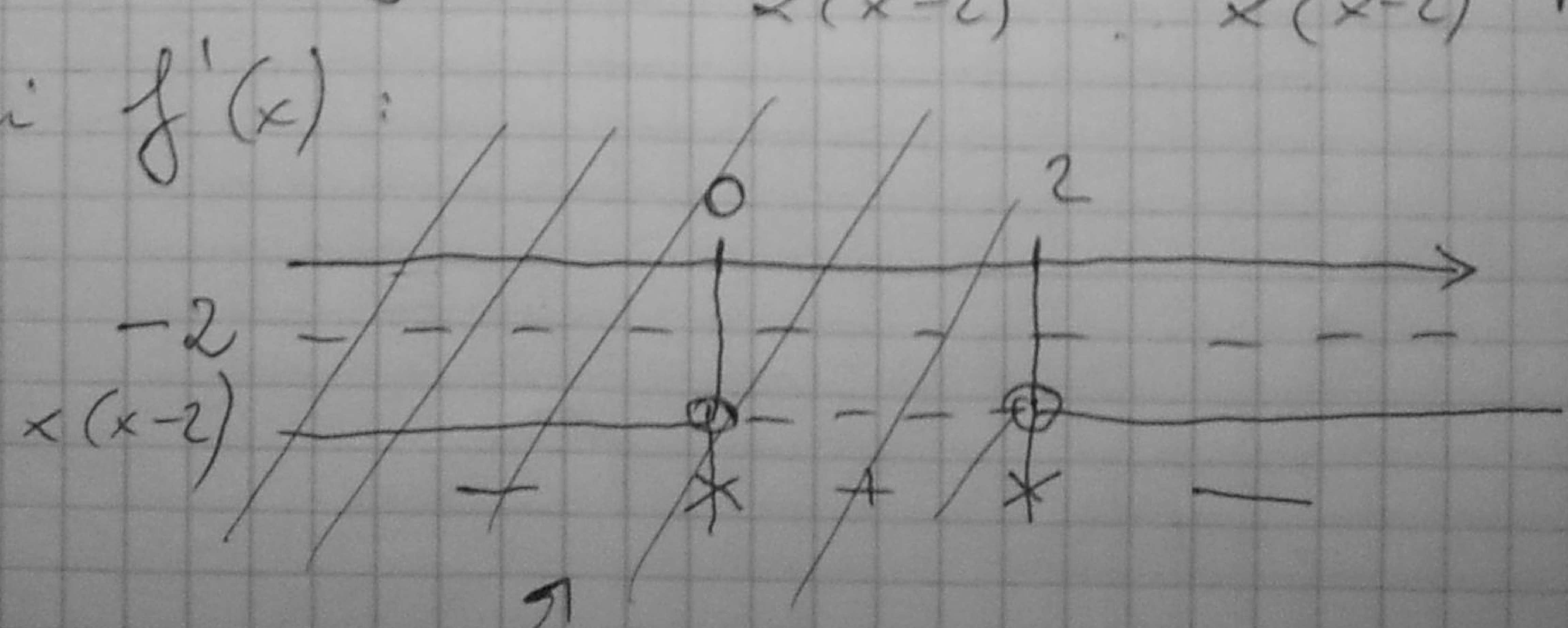
$\Rightarrow \lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \ln \frac{x}{x-2} = \ln 1 = 0,$

(perché $\ln x$ è una funzione CONTINUA, e quindi $\lim \ln \frac{x}{x2} = \ln\left(\lim \frac{x}{x-2}\right)$)

$\Rightarrow$ ASINTOTO ORIZZONTALE $y = 0$

d) •) $f'(x) = \frac{1}{x} - \frac{1}{x-2} = \frac{x - 2 - x}{x(x-2)} = \frac{-2}{x(x-2)}$ per $x > 2$

Segno di $f'(x)$:




NON esiste $f$, $\Rightarrow$ NON esiste $f'$

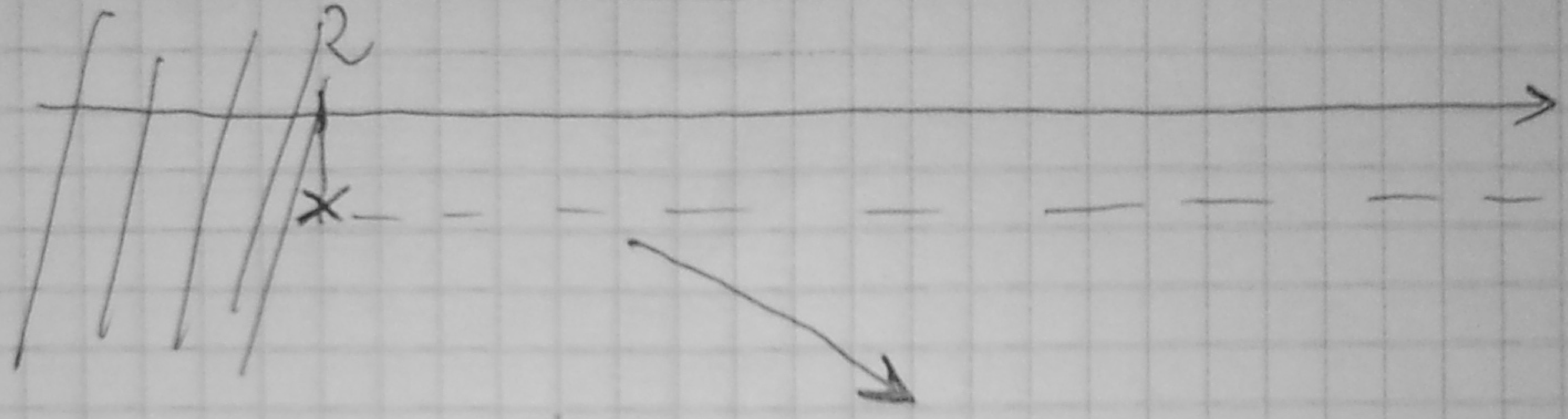
•) Segno $f'$, METODO ALTERNATIVO:

$$f'(x) = \frac{-2}{x\,(x-2)} \quad \text{per } x>2.$$

Poichè $x>2$, abbiamo al denominatore D

$x>2>0$ e $x-2>0$ $\Rightarrow$ $D>0$ $\Rightarrow$ $f'(x)<0$

$\Rightarrow$



e) $f(x)$ è decrescente, e parte da $+\infty$ $\left(\lim_{x\to 2^+}\right)$ per arrivare a $0$ $\left(\lim_{x\to+\infty}\right)$ $\Rightarrow$ non ha MAX o MIN di nessun tipo. Si ha

$\sup f = +\infty$, $\inf f = 0$.

f) $f'(x) = \frac{1}{x} - \frac{1}{x-2}$ $\Rightarrow$

$$\Rightarrow f''(x) = -\frac{1}{x^2} + \frac{1}{(x-2)^2} = \frac{-(x-2)^2+x^2}{x^2(x-2)^2}$$

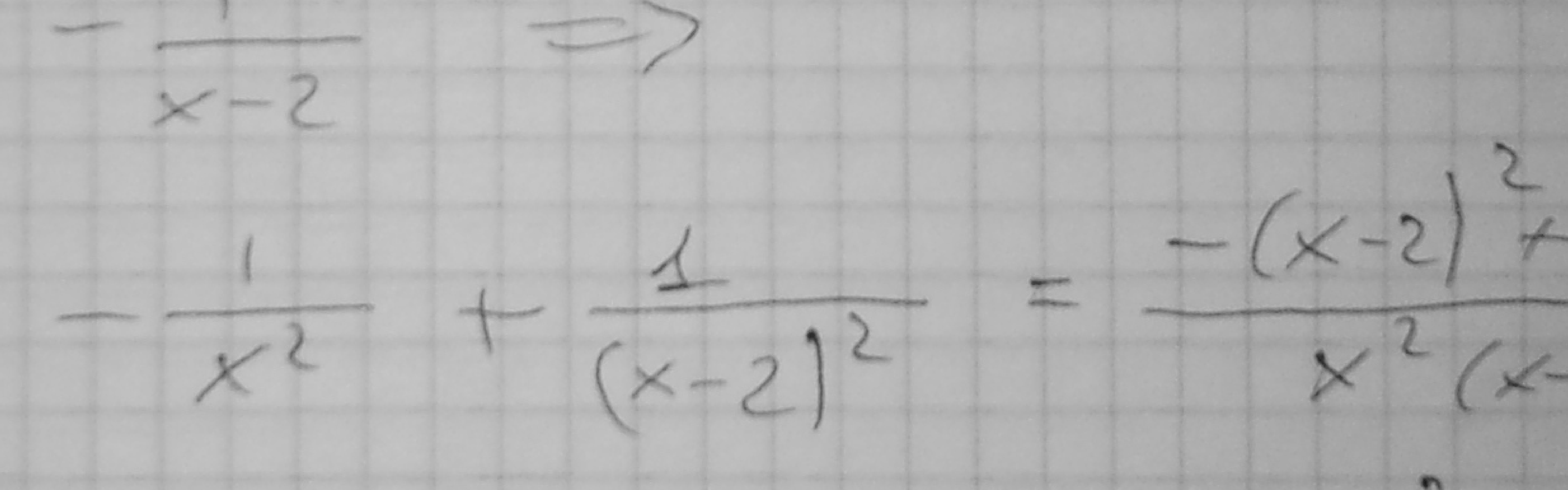
Segno di $f''(x)$ = segno di $x^2-(x-2)^2 =$

$= (x-x+2)(x+x-2) = 2(2x-2)$

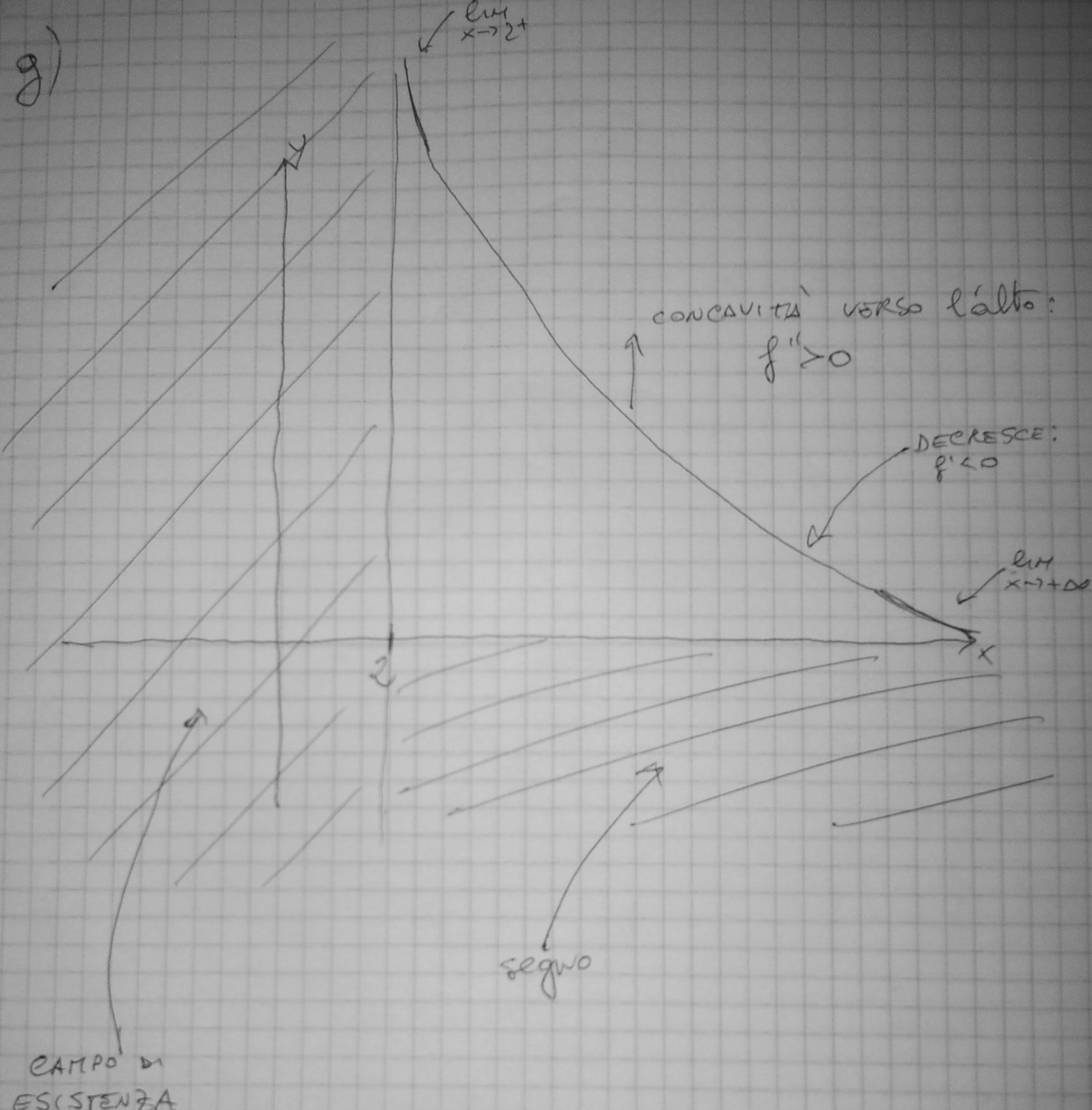
= segno di $x-1$.

Ma $x-1>0$ $\forall x \in (2,+\infty)$

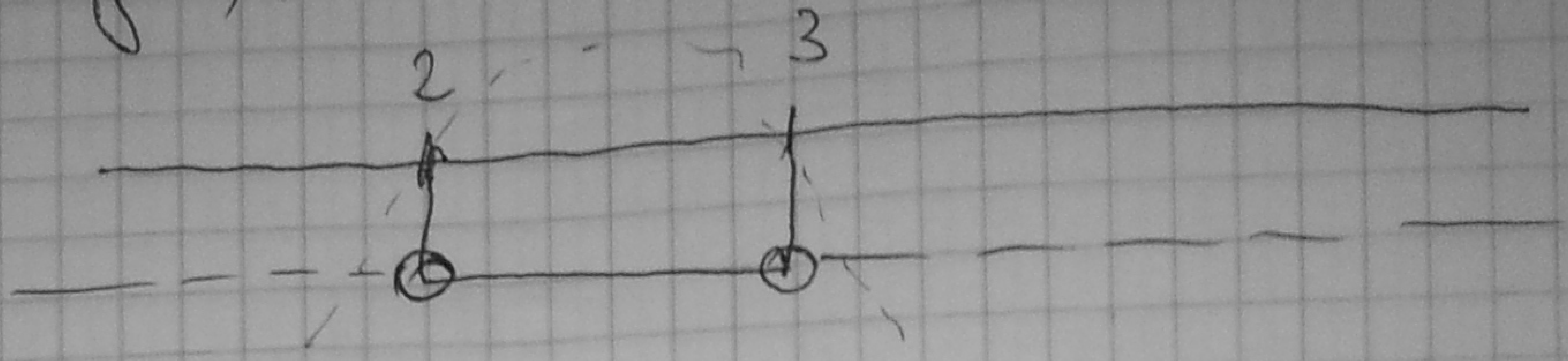
$\Rightarrow f''(x)>0$ $\forall x\in(2,+\infty)$

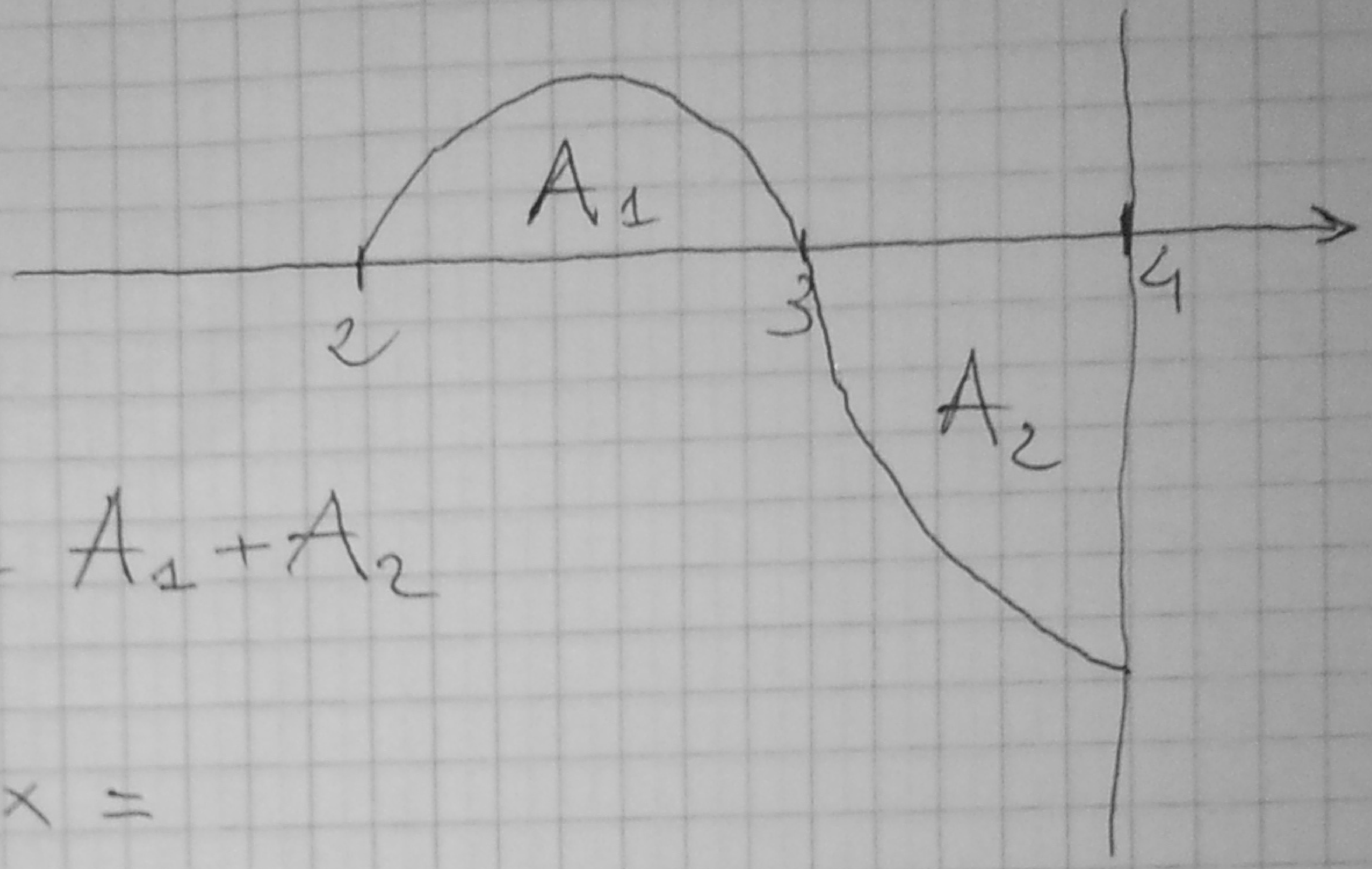


g)


lim x→2+
y
CONCAVITÀ VERSO L'ALTO:
f''>0
DECRESCE:
f'<0
lim x→+∞
x
2
segno
CAMPO DI
ESISTENZA

4) Segno di $f(x)$:




Area tra 2 e 4 = $A_1 + A_2$

$$A_1 = \int_2^3 (3-x)(x-2)\,dx =$$

$$= \int_2^3 (-x^2+5x-6)\,dx = \left[-\frac{x^3}{3} + \frac{5x^2}{2} - 6x\right]_2^3 =$$

$$= -9 + \frac{45}{2} - 18 - \left(-\frac{8}{3} + 10 - 12\right) = -25 + \frac{135+16}{6}$$

$$= -25 + \frac{151}{6} = \frac{1}{6}$$ ( >0 , quindi abbiamo speranza che sia GIUSTO)

$$A_2 = \int_3^4 (x^2-5x+6)\,dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right]_3^4 =$$

$$= \frac{64}{3} - 40 + 24 - \left(9 - \frac{45}{2} + 18\right) =$$

opposti, quindi probabilmente GIUSTO

$$= -43 + \frac{128+135}{6} = -43 + \frac{263}{6} = \frac{-258+263}{6} = \frac{5}{6} > 0$$

$$\Rightarrow \boxed{A = A_1 + A_2 = \frac{1}{6} + \frac{5}{6} = 1}$$

5) $\displaystyle\int_2^3 \frac{3x^2-4}{\sqrt[3]{x^3-4x}}\,dx$

Poiché $\sqrt[3]{x^3-4x}\Big|_2 = 0$, l'INTEGRALE è IMPROPRIO.

CALCOLIAMO LA PRIMITIVA:

$$\int \frac{3x^2-4}{\sqrt[3]{x^3-4x}}\,dx = \int \frac{1}{\sqrt[3]{x^3-4x}}\,d\left(3\frac{x^3}{3}-4x\right) =$$

$$\overset{t=x^3-4x}{=} \int \frac{1}{\sqrt[3]{t}}\,dt = \int t^{-1/3}\,dt = \frac{3}{2}t^{2/3} =$$

$$= \boxed{\frac{3}{2}\left(x^3-4x\right)^{2/3}}$$

Verifica: $\left(\frac{3}{2}\left(x^3-4x\right)^{2/3}\right)' = \frac{3}{2}\,\frac{2}{3}\left(x^3-4x\right)^{-1/3}\cdot\left(3x^2-4\right)$

$$= \frac{1}{\sqrt[3]{x^3-4x}}\left(3x^2-4\right) \qquad \boxed{\text{ok}}$$

$$\Rightarrow \int_2^3 \frac{3x^2-4}{\sqrt[3]{x^3-4x}}\,dx = \lim_{t\to 2^+}\left[\frac{3}{2}\left(x^3-4x\right)^{2/3}\right]_t^3 =$$

$$= \frac{3}{2}\sqrt[3]{225} - \frac{3}{2}\underbrace{\lim_{t\to 2^+}\left(t^3-4t\right)^{2/3}}_{=0} = \boxed{\frac{3}{2}\sqrt[3]{225}}$$

6) $\sum_{1}^{+\infty} \frac{3+n}{1+3n}$ $\boxed{\text{NON CONVERGE}}$, perchè $\lim_{n\to+\infty} \frac{3+n}{1+3n} = \frac{1}{3} \neq 0$

$$\sum_{1}^{+\infty} 4^{n+1} \cdot 3^{-2n} = \sum_{1}^{+\infty} \frac{4^{n+1}}{9^n} = 4\sum_{1}^{+\infty} \left(\frac{4}{9}\right)^n$$

$\boxed{\text{CONVERGE}}$, perchè serie geometrica di ragione $<1$

Abbiamo $4\sum_{1}^{+\infty} \left(\frac{4}{9}\right)^n = 4\left[\frac{1}{1-4/9} - 1\right] = \boxed{\frac{16}{5}}$

7) 1) $\det A = +4+2+1 = 7 \neq 0$

2) $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = B$ AMMETTE UNA e UNA SOLA SOLUZIONE, perchè $\det A \neq 0$. RISOLVO CON CRAMER:

$$x_1 = \frac{\det\begin{pmatrix} 0 & 0 & -1 \\ -1 & 1 & 0 \\ -2 & 2 & 2 \end{pmatrix}}{7} = \frac{-4}{7}$$

$$x_2 = \frac{\det\begin{pmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}}{7} = \frac{2-2+1}{7} = \frac{1}{7}$$

$$x_3 = \frac{\det\begin{pmatrix} 2 & 0 & 0 \\ -1 & 1 & 1 \\ 1 & 2 & -2 \end{pmatrix}}{7} = \frac{-8}{7}$$

<u>Verifica:</u>

$$\begin{pmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} -4/7 \\ 1/7 \\ -8/7 \end{pmatrix} = \begin{pmatrix} -\frac{8}{7}+\frac{8}{7} \\ 4/7+\frac{1}{7} \\ -4/7+2/7-\frac{16}{7} \end{pmatrix} = \begin{pmatrix} 0 \\ 5/7 \\ -\frac{18}{7} \end{pmatrix}$$

$\neq B \Rightarrow$ C'È UN ERRORE. CERCHIAMOLO.

$\Rightarrow$

$\Rightarrow$ CONTROLLO $\det A = 7$ ✓

CONTROLLO $x_1$:

$$x_1 = \frac{\det\begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 0 \\ -2 & 2 & 2 \end{pmatrix}}{7} = \frac{-2-2}{7} = -\frac{4}{7} \checkmark$$

ERA SBAGLIATO

CONTROLLO $x_2$:

$$x_2 = \frac{\det\begin{pmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}}{7} = \frac{4-2+1}{7} = \frac{3}{7}$$

CONTROLLO $x_3$

$$x_3 = \frac{\det\begin{pmatrix} 2 & 0 & 0 \\ -1 & 1 & 1 \\ 1 & 2 & -2 \end{pmatrix}}{7} = \frac{-4-4}{7} = \frac{-8}{7} \checkmark$$

VERIFICA

$$\begin{pmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} -\frac{4}{7} \\ \frac{3}{7} \\ -\frac{8}{7} \end{pmatrix} = \begin{pmatrix} -\frac{8}{7} + \frac{8}{7} \\ \frac{4}{7} + \frac{3}{7} \\ -\frac{4}{7} + \frac{6}{7} - \frac{16}{7} \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ \frac{7}{7} \\ -\frac{14}{7} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} = B \checkmark$$

$$\Rightarrow \boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4/7 \\ 3/7 \\ -8/7 \end{pmatrix}}$$

7.3) Rango $A = 3$, perchè $\det A \neq 0$

Rango $B = 1$, perchè c'è almeno 1 elemento $\neq 0$.